Penney Ante Game

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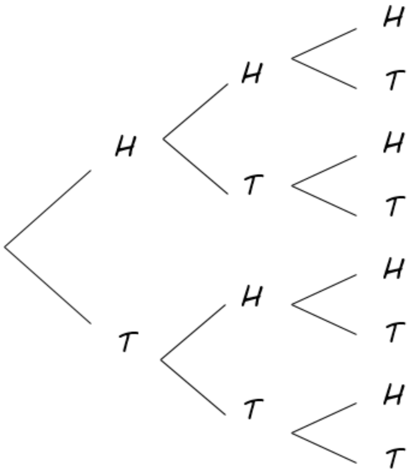

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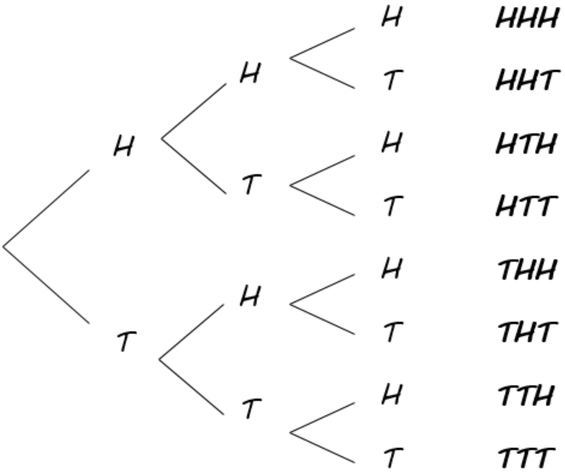

The Penney Ante Game

The Game:

- Penney Ante is a game played between two players, A and B.
- Player A chooses a sequence of three consecutive coin flips.
- After Player A has chosen, only then does Player B choose a sequence of three coin flips.
- The player whose sequence occurs first wins.

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Instructions:

- One player should be in charge of flipping the coin.
- The other player records the sequence on the game sheet.
- Play 5 rounds and then switch the role of Player A and Player B.

Game	Player Selection	Sequence	Winner
1.	Player A: H H H Player B: T H H	HTHTTHTHH	A: ☐ B: ☒

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Player B is able to win more often, since they have the opportunity to counter-pick a sequence in response to what Player A chooses.

The best way to increase your chances of winning as Player B is to change the second pick of Player A and move it to the front of their sequence.

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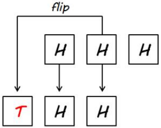
If Player A picks: [1] [2] [3]
Player B's picks: [not-2] [1] [2]



For example, if Player A picks HHH, then Player B should pick THH.

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What are the chances?

A's Choice	B's Choice	Probability of B Winning	Odds in Favour of B
HHH			
HHT			
HTH			
HTT			
THH			
THT			
TTH			
TTT			

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THT	TTH		
TTH	HTT		
TTT	HTT		

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A's Choice	B's Choice	Probability of B Winning	Odds in Favour of B
HHH	THH	$\frac{7}{8}$	
HHT	THH	$\frac{3}{4}$	
HTH	HHT	$\frac{2}{3}$	
HTT	HHT	$\frac{2}{3}$	
THH	TTH	$\frac{2}{3}$	
THT	TTH	$\frac{2}{3}$	
TTH	HTT	$\frac{3}{4}$	
TTT	HTT	$\frac{7}{8}$	

What are the chances?

A's Choice	B's Choice	Probability of B Winning	Odds in Favour of B
HHH	THH	$\frac{7}{8}$	7:1
HHT	THH	$\frac{3}{4}$	3:1
HTH	HHT	$\frac{2}{3}$	2:1
HTT	HHT	$\frac{2}{3}$	2:1
THH	TTH	$\frac{2}{3}$	2:1
THT	TTH	$\frac{2}{3}$	2:1
TTH	HTT	$\frac{3}{4}$	3:1
TTT	HTT	$\frac{7}{8}$	7:1

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When we count normally, each place value represents lots of powers of 10. eg.

$$321 = 3 \times 10^2 + 2 \times 10^1 + 1 \times 10^0$$

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Binary counts in base 2 which uses lots of powers of 2.
eg.

$$\begin{aligned} 101_2 &= 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \\ &= 5 \end{aligned}$$

Conway's Algorithm

This uses comparisons between Player A and Player B's sequences and assigns binary values.

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How to perform it:

- Place the sequences one above the other with aligned digits.
- Compare the whole of the two sequences.
- If they are the same, put 1 above the first term, if not put a 0.
- Remove the leading term from the upper sequence and shift to the left.
- Compare the first two digits.
- If they are the same, put 1 above the leading element, otherwise a 0.
- Repeat the shift and check for the last element.
- Compile the results as a 3 digit binary number.

Conway's Algorithm (cont'd)

Using the above steps, compare AA, AB, BB & BA. After converting the binary values, substitute into the following expression:

$$\frac{AA - AB}{BB - BA}$$

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eg. when A is given by HHH and B is given by THH:

$$\begin{aligned} AA &= 111_2 \\ AB &= 000_2 \\ BB &= 100_2 \\ BA &= 011_2 \end{aligned}$$

Converting and substituting into the above gives:

$$\frac{7 - 0}{4 - 3} = 7$$

$\therefore$ Player B's odds are 7, which as a ratio is 7:1.